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## Projection and Resolution of Vectors

Def ${ }^{\underline{n}}$ Let A be a non-zero vector. The projection of a vector B onto A, denoted by $\operatorname{Proj}_{A} B$ is defined as:

$$
\operatorname{Pr} o j_{A} B=\frac{A \cdot B}{|A|^{2}} A
$$

Note that: $\operatorname{Pr} o j_{A} B$ is a vector parallel to A .

Example: Let $\mathrm{A}=(-2,-3,-1)$ and $\mathrm{B}=(0,1,-1)$. Find $\operatorname{Proj}{ }_{A} B$ and $\operatorname{Proj}_{B} A$.

Solution: $|A|=\sqrt{14},|B|=\sqrt{2}$ and $A \cdot B=-2$.
Therefore, $\operatorname{Proj}{ }_{A} B=-\frac{1}{7}$ A and $\operatorname{Proj}{ }_{B} A=-\mathrm{B}$.

## Theorem:

Let A be a non-zero vector. Then for any vector B,

$$
\left|\operatorname{Proj}_{A} B\right| \leq|B|
$$

Proof: $\left|\operatorname{Proj}_{A} B\right|=\frac{|A \cdot B|}{|A|}=|B||\cos \theta| \leq|B|$.
Therefore $\left|\operatorname{Proj}_{A} B\right| \leq|B|$.

Now let A and B be orthogonal vectors and let C be a vector in the same plane as A and B .
Then we can express $C$ as a linear combination of vectors parallel to $A$ and $B$ as follows:

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\mathrm{C}=\operatorname{Pr} o j_{A} C+\operatorname{Pr} o j_{B}^{C}
$$

In this case, we say that vector $C$ is resolved into vectors parallel to $A$ and $B$.

Example: Let $\mathrm{A}=(0,1,-2), \mathrm{B}=(0,2,1)$ and $\mathrm{C}=(0,5,-4)$. Resolve C into vectors parallel to A and B .

Solution: $A \cdot B=0$ and these three vectors lie on the yz plane.

$$
\operatorname{Proj}_{A} C=\frac{A \cdot C}{|A|^{2}} A \text { and } \operatorname{Proj}_{B} C=\frac{B \cdot C}{|B|^{2}} B
$$

Now $A \cdot C=13, B \cdot C=6,|A|=|B|=\sqrt{5}$.
Hence $\operatorname{Proj}{ }_{A} C=\frac{13}{5} A$ and $\operatorname{Proj}{ }_{B} C=\frac{6}{5} B$.

Therefore $\mathrm{C}=\frac{13}{5} A+\frac{6}{5} B$.
Example: Let $A=(1,0,3), B=(-3,0,1)$ and $C=(2,0,5)$. Resolve $C$ into vectors parallel to $A$ and $B$.

Solution: $A \cdot B=0$ and these three vectors lie on the xz plane.

$$
\operatorname{Proj}_{A} C=\frac{A \cdot C}{|A|^{2}} A \text { and } \operatorname{Proj} B=\frac{B \cdot C}{|B|^{2}} B
$$

Now $A \cdot C=18, B \cdot C=-1,|A|=|B|=\sqrt{10}$.
Hence $\operatorname{Proj}{ }_{A} C=\frac{9}{5} A$ and $\operatorname{Proj}{ }_{B} C=-\frac{1}{10} B$.

Therefore, $\mathrm{C}=\frac{9}{5} A-\frac{1}{10} B$.

