C4.2. We have seen that the direct method of signal modeling leads to a set of nonlinear equations that need to be solved for the model coefficients. Iterative prefiltering, however, is an approach that may be used to avoid having to solve these nonlinear equations. In this exercise we look at the method of iterative prefiltering and compare it to Prony's method.

(a) Let

$$H(z) = \frac{1 - 0.9z^{-1} + 0.81z^{-2}}{1 - 1.978z^{-1} + 2.853z^{-2} - 1.877z^{-3} + 0.9036z^{-4}}$$

be the system function of a linear shift-invariant system. Generate the first 100 samples of the unit sample response $h(n)$ of this filter.

(b) Using the method of iterative prefiltering, find a two-zero, four-pole model for $h(n)$. How many iterations are required for the coefficients to converge? What happens if $h(n)$ is over-modeled using $p = q = 4$? What about $p = q = 5$?

(c) The model found in part (b) assumes exact measurements of the unit sample response $h(n)$. Suppose that the measurements of $h(n)$ are noisy and you observe

$$y(n) = h(n) + v(n)$$

where $v(n)$ is white Gaussian noise with a variance σ_v^2. Repeat part (b) using these noisy measurements with $\sigma_v^2 = 0.0001, 0.001, 0.01$. Comment on the accuracy of your models and the sensitivity of the coefficients to the noise variance.

(d) Repeat parts (b) and (c) using Prony's method and compare your results with those obtained using iterative prefiltering. Which method works the best?