C7.3. The state estimation equation in the discrete Kalman filter is

$$\hat{\mathbf{x}}(n|n) = \mathbf{A}(n-1)\hat{\mathbf{x}}(n-1|n-1) + \mathbf{K}(n)\Big[\mathbf{y}(n) - \mathbf{C}(n)\mathbf{A}(n-1)\hat{\mathbf{x}}(n-1|n-1)\Big]$$

Thus, given the state transition matrix $\mathbf{A}(n)$ and the observation matrix $\mathbf{C}(n)$, all that is required is the Kalman gain $\mathbf{K}(n)$. Since the Kalman gain does not depend upon the state $\mathbf{x}(n)$ or the observations $\mathbf{y}(n)$, the Kalman gain may be computed off-line prior to filtering.

(a) Write a MATLAB program `gain.m` to compute the Kalman gain $\mathbf{K}(n)$ for a stationary process with

$$\begin{aligned} \mathbf{x}(n) &= \mathbf{A}\mathbf{x}(n-1) + \mathbf{w}(n) \\ \mathbf{y}(n) &= \mathbf{C}\mathbf{x}(n) + \mathbf{v}(n) \end{aligned}$$

(b) Suppose that $x(n)$ is a third-order autoregressive process

$$x(n) = -0.1x(n-1) - 0.09x(n-2) + 0.648x(n-3) + w(n)$$

where $w(n)$ is unit variance white noise, and that the observations are

$$y(n) = x(n) + v(n)$$

where $v(n)$ is white noise with a variance $\sigma_v^2 = 0.64$. What initialization should you use for $P(0|0)$? Using this initialization, find the Kalman gain $\mathbf{K}(n)$ for $n = 0$ to $n = 10$.

(c) What is the steady-state value for the Kalman gain? How is it affected by the initialization $P(0|0)$?

(d) Generate the processes $x(n)$ and $y(n)$ in part (b) and use your Kalman filter to estimate $x(n)$ from $y(n)$. Plot your estimate and compare it to $x(n)$.

(e) Repeat parts (b) and (d) for the process

$$x(n) = -0.95x(n-1) - 0.9025x(n-2) + w(n) - w(n-1)$$

where $w(n)$ is unit variance white noise, and the observations are

$$y(n) = x(n) + v(n)$$

where $v(n)$ is white noise with a variance $\sigma_v^2 = 0.8$.